

Electroweak Radiative Corrections to Weak Boson Production at Hadron Colliders

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3. Electroweak Corrections to W Production
4. Future Plans
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1 – Introduction

- Precise measurements have to be matched by precise theoretical predictions
- Expectations for electroweak measurements in Run II of the Tevatron:
 - ➡ $\delta M_W \approx 40 \text{ MeV}$ per channel and experiment for 2 fb^{-1}
 - ➡ $\delta \Gamma_W \approx 50 \text{ MeV}$ per channel and experiment for 2 fb^{-1} from tail of transverse mass distribution
 - ➡ $\delta \sin^2 \theta_W \approx 6 \times 10^{-4}$ per channel and experiment for 10 fb^{-1}
 - ➡ W/Z cross section ratio, \mathcal{R} , to $\approx 0.5\%$ (extract Γ_W)
- use σ_W as a luminosity monitor
- most important of these measurements: M_W
 - ➡ together with m_{top} determines indirect bounds on Higgs boson mass

- For these measurements, it is necessary to **fully** understand QCD **and** EWK radiative corrections to W and Z production
- QCD corrections: in good shape
 - ☞ $\mathcal{O}(\alpha_s^2)$ for cross section
 - ☞ resummed W and Z p_T distributions are known
- **EWK corrections**
 - ☞ electroweak corrections **shift W and Z masses by $\mathcal{O}(100 \text{ MeV})$**
 - ☞ same for Γ_W from tail of transverse mass (M_T) distribution
 - ☞ most of the effect comes from final state photon radiation
 - ☞ need to understand EWK corrections for W **and** Z production:
 - ➔ Measuring M_Z and Γ_Z helps to calibrate detector

- (< 1997) (Berends, Kleiss (1985))

☞ only final state corrections taken into account

☞ soft and virtual $\mathcal{O}(\alpha)$ corrections are estimated indirectly from the $\mathcal{O}(\alpha^2)$ $W \rightarrow \ell\nu\gamma$, $Z \rightarrow \ell^+\ell^-\gamma$ width and the hard photon contribution

☞ CDF's and DØs guess-timate of uncertainty from unknown EWK corrections in Run I analyses:

$$\delta M_W \approx 20 \text{ MeV}$$

- recent developments:

- ☞ full $\mathcal{O}(\alpha)$ QED corrections to Drell-Yan (Z) production (UB, S. Keller, W.K. Sakumoto)

- ☞ full $\mathcal{O}(\alpha)$ electroweak corrections to Drell-Yan (Z) production (UB, O. Brein, W. Hollik, C. Schappacher, D. Wackeroth)

- ☞ $\mathcal{O}(\alpha)$ electroweak corrections to W production in the pole approximation (UB, S. Keller, D. Wackeroth)

- ☞ full $\mathcal{O}(\alpha)$ electroweak corrections to W production (S. Dittmaier, M. Krämer and UB, D. Wackeroth, in preparation)

2 – Electroweak Corrections to Drell-Yan Production

- For Drell-Yan production, the 1-loop EWK corrections can be split into separately gauge invariant subsets of diagrams:
 - ☞ QED corrections
 - initial state QED corrections
 - final state QED corrections
 - ☞ purely weak corrections
- consider only QED corrections for the moment (UB, S. Keller, W. Sakamoto)
- employ NLO Monte Carlo technique for calculation (recent review: Harris and Owens, PRD 65, 094032 (2002))
 - ☞ isolate soft and collinear singularities associated with real photon emission.

☞ partition phase space into soft, collinear and finite regions by introducing theoretical cutoffs δ_s and δ_c

☞ for

$$E_\gamma < \delta_s \frac{\sqrt{\hat{s}}}{2}$$

evaluate $2 \rightarrow 3$ diagrams in soft photon approximation ($\sqrt{\hat{s}}$: parton CM energy)

☞ soft singularities from final state radiation (FSR) cancel against those from interference of Born and virtual final state corrections

☞ the same applies to initial state radiation (ISR) and interference effects

☞ for

$$E_\gamma > \delta_s \frac{\sqrt{\hat{s}}}{2}$$

use full $2 \rightarrow 3$ matrix elements. Evaluate via Monte Carlo.

- Collinear singularities

- ☞ Final state collinear singularities are regulated by finite lepton masses

- ☞ Initial state collinear singularities are **universal to all orders** and can be absorbed into the parton distribution functions (PDF's), in complete analogy to QCD

- Evaluate matrix elements for

$$|\hat{t}|, |\hat{u}| < \delta_c \hat{s}$$

(\hat{t}, \hat{u} : standard Mandelstam variables) in leading pole approximation

- ☞ factorize singularities into PDF's

- ☞ Evaluate remainder as part of $2 \rightarrow 2$ contribution

- ☞ for

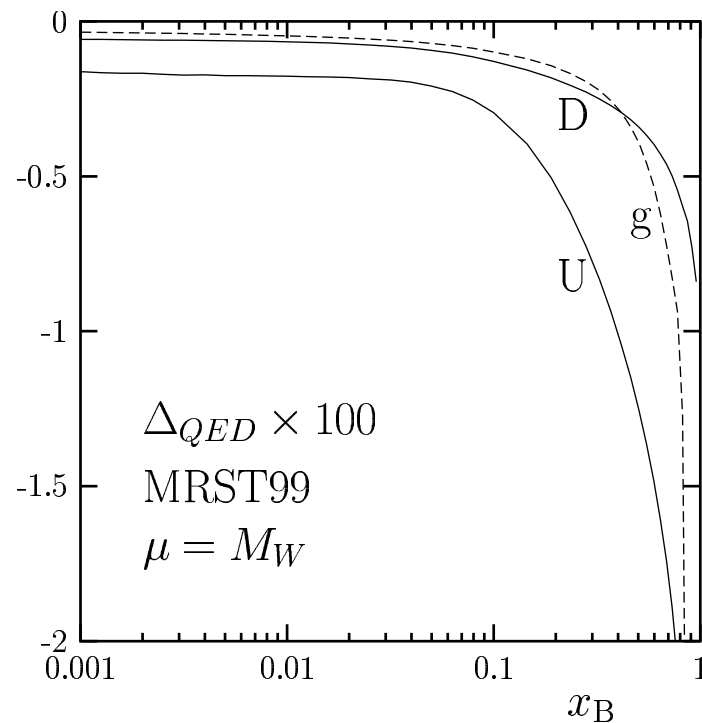
$$|\hat{t}|, |\hat{u}| > \delta_c \hat{s}$$

evaluate full $2 \rightarrow 3$ matrix element

→ for a consistent treatment of the $\mathcal{O}(\alpha)$ initial state corrections, QED corrections should be incorporated into the global fitting of PDF's.

👉 need QED corrections to PDF's

👉 QED corrections to PDF's are small except at large x (**Spiesberger**)



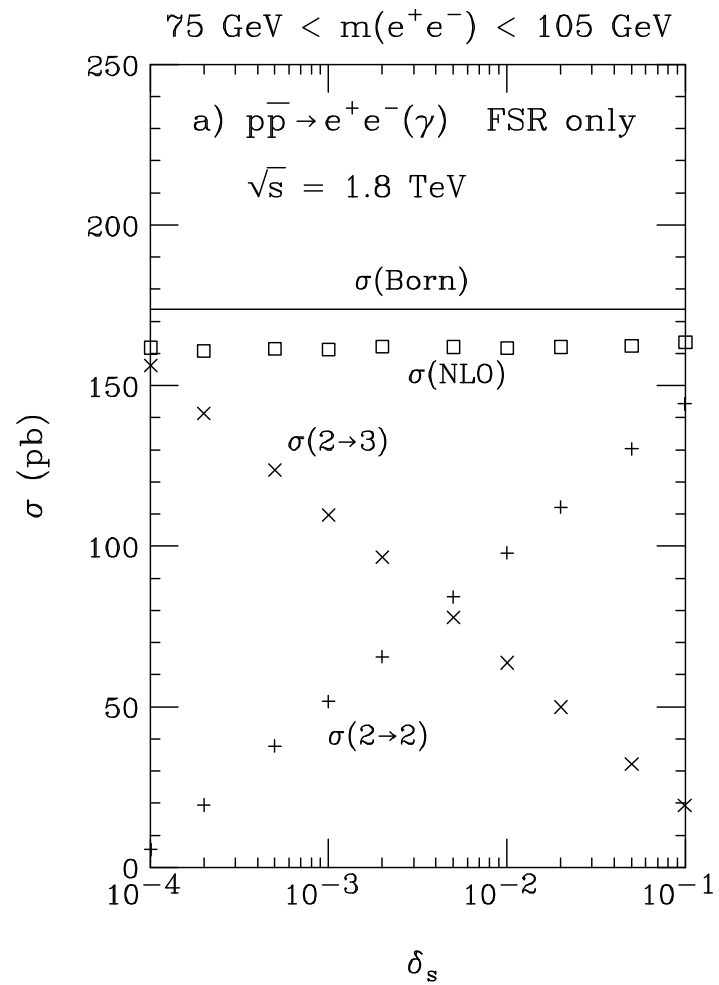
$$[U = \sum_{gen}(u + \bar{u}), D = \sum_{gen}(d + \bar{d})]$$

- 👉 also need QED corrections for all data sets used to fit PDF's
- 👉 Absorbing the collinear singularities into the PDF's introduces a QED factorization scheme dependence
- 👉 we performed our calculation in the QED $\overline{\text{MS}}$ and QED DIS schemes
- 👉 current global fits to the PDF's do not take into account QED corrections
- strictly speaking our calculation is incomplete
- 👉 fortunately initial state corrections are small

- final result

- 👉 two sets of weighted events corresponding to $2 \rightarrow 2$ and $2 \rightarrow 3$ contributions
- 👉 each set depends on δ_s and δ_c
- 👉 their sum must be independent of δ_s and δ_c (as long as these parameters are sufficiently small so that the soft photon and pole approximations hold)

Example



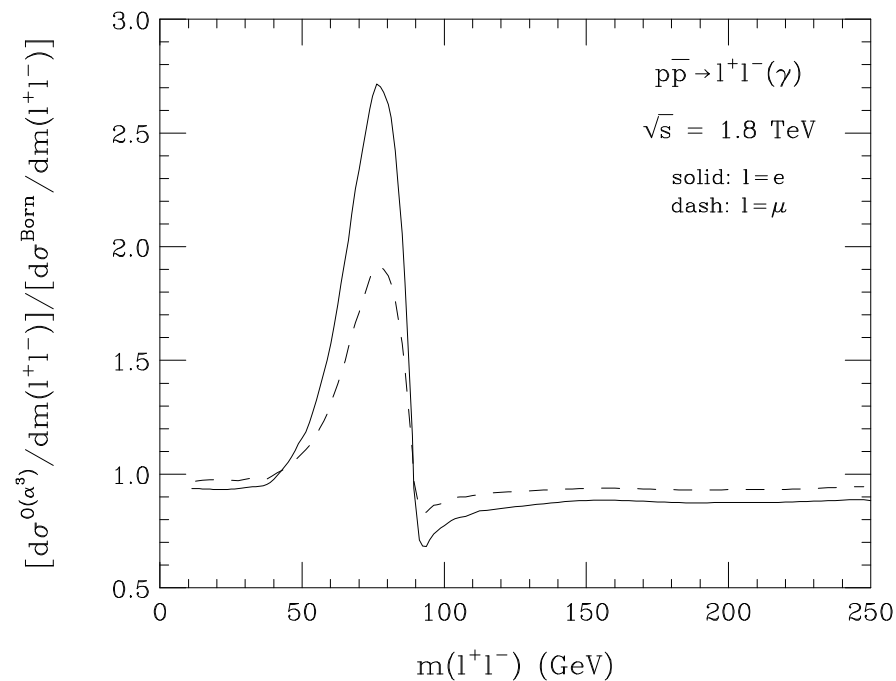
- phenomenological results

☞ FSR terms are proportional to

$$\frac{\alpha}{\pi} \log \left(\frac{\hat{s}}{m_\ell^2} \right)$$

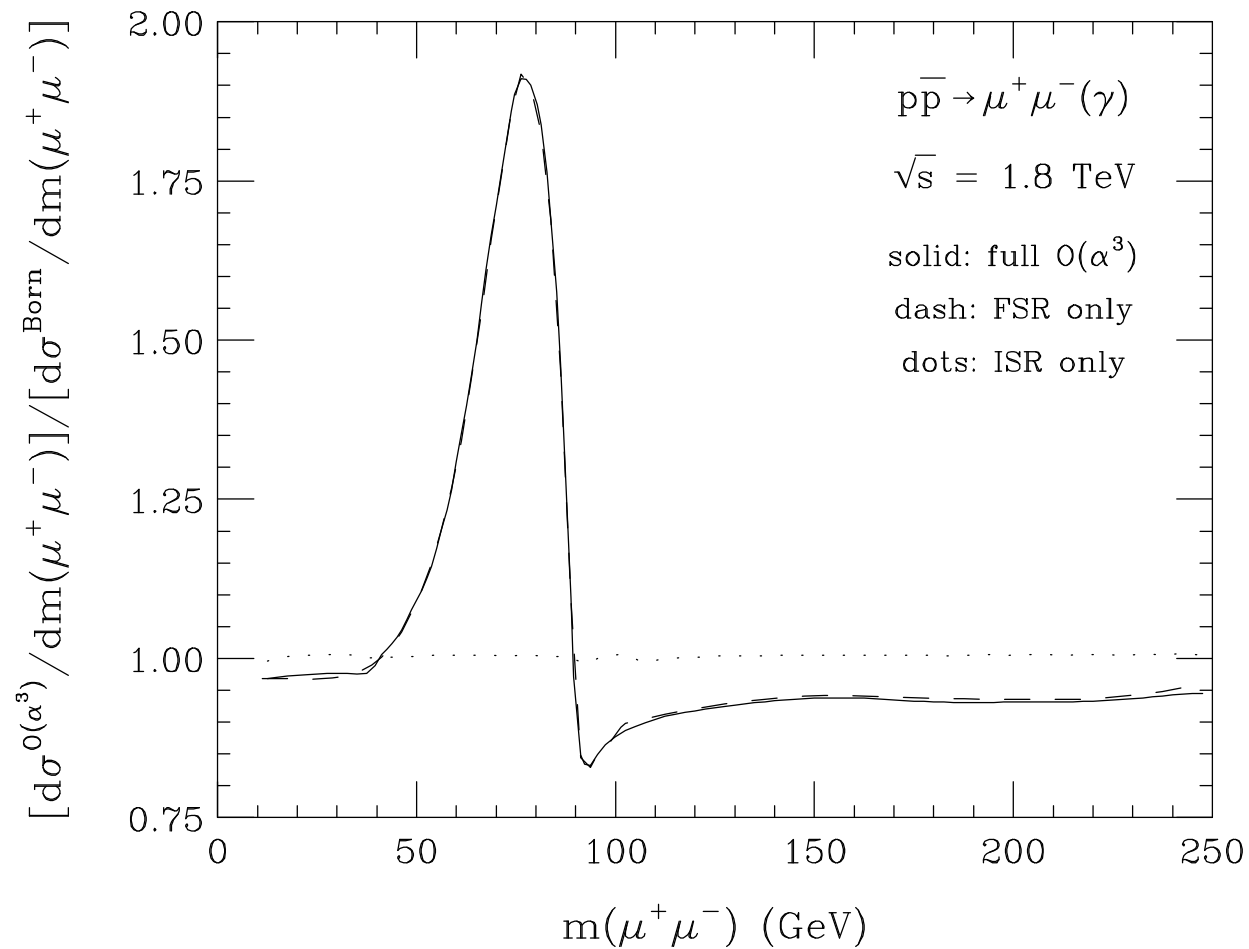
→ these terms significantly influence the $\ell^+ \ell^-$ inv. mass distribution

☞ Tevatron:



- big enhancement below the peak (due to Breit Wigner peak)
- at the peak the cross section is reduced by about 30% for electrons and 20% for muons
- for $m(\ell\ell) > 120$ GeV, the cross section is reduced by about $\sim 12\%$ ($\sim 7\%$) for e (μ)
- integrating over $m(\ell\ell)$, the large positive and negative corrections cancel (KLN theorem)

- final state corrections dominate everywhere
- Initial state corrections are small and uniform



- Experimental lepton ID and QED corrections

- ☞ Detector effects may significantly influence the QED corrections

- ☞ to be specific use simple model of run I CDF detector

- ☞ acceptance cuts:

$$p_T(e) > 20 \text{ GeV} \quad |\eta(e)| < 2.4$$

$$p_T(\mu) > 25 \text{ GeV} \quad |\eta(\mu)| < 1.0$$

require at least one e (μ) with $|\eta(e)| < 1.1$ ($|\eta(\mu)| < 0.6$)

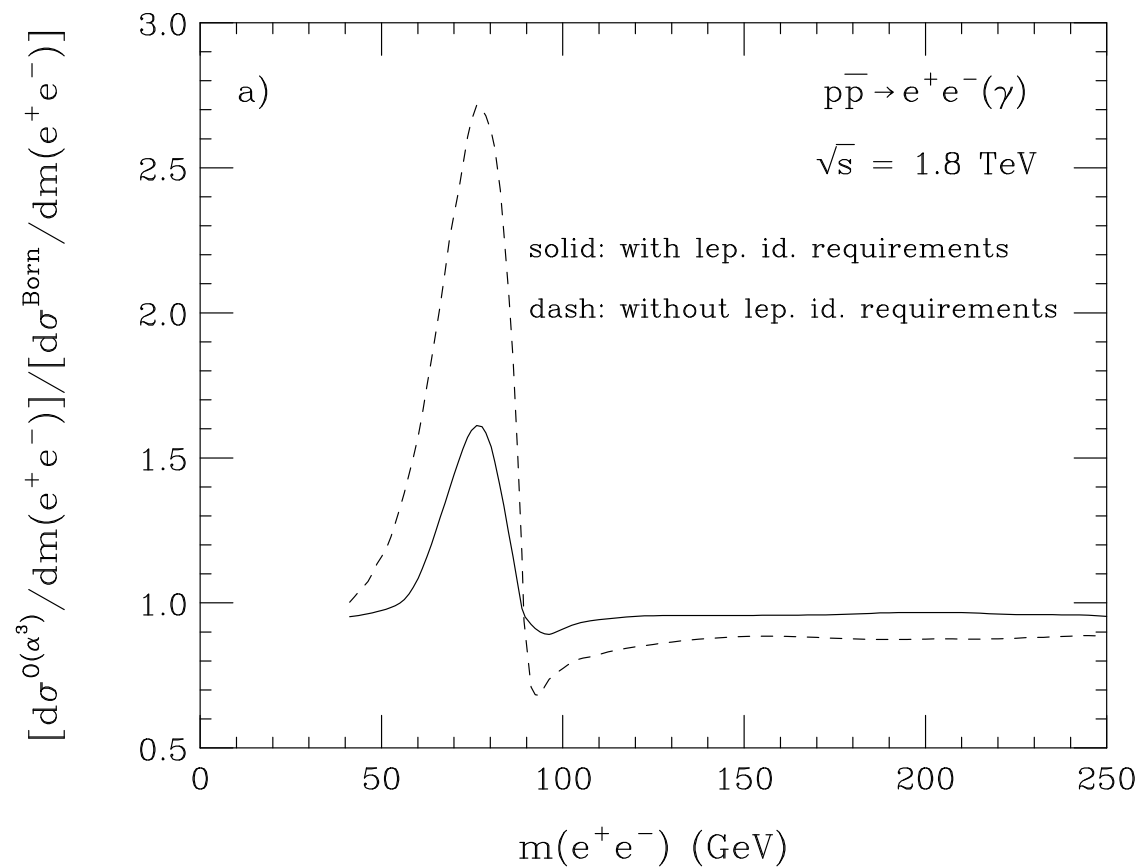
- ☞ smear momenta according to CDF resolution

- ☞ assume calorimeter segmentation of $\Delta\eta \times \Delta\Phi = 0.1 \times 15^\circ$

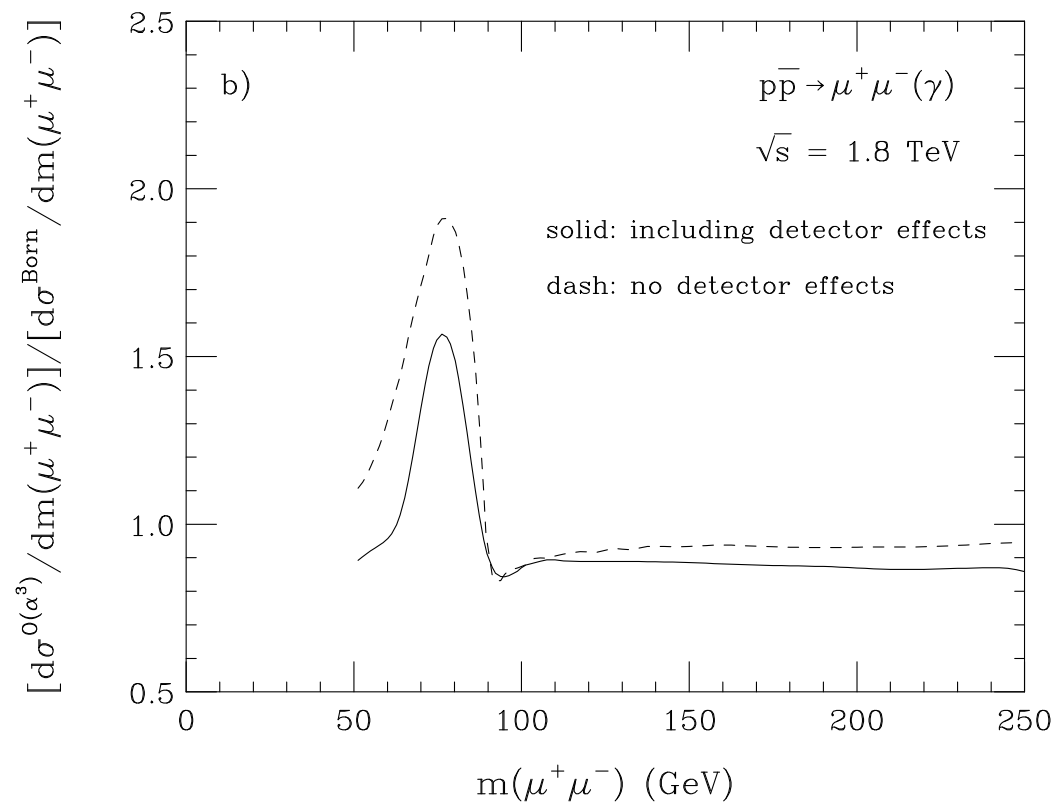
- It is difficult to discriminate electrons and photons which hit the same calorimeter cell

- recombine e and γ momenta to an effective electron momentum in that case

- an inclusive quantity is formed
- the mass singular terms $((\alpha/\pi) \log(\hat{s}/m_\ell^2))$ disappear (KLN again...)
- the effect of the QED corrections is reduced



- Muons must be consistent with a minimum ionizing particle
 - require $E_\gamma < 2 \text{ GeV}$ in cell traversed by muon
 - this reduces the hard photon part
 - the mass singular terms survive



- Impact of radiative corrections on A_{FB}

☞ Define

$$A_{FB} = \frac{F - B}{F + B}$$

$$F = \int_0^1 \frac{d\sigma}{d\cos\theta^*} d\cos\theta^*, \quad B = \int_{-1}^0 \dots$$

$$\cos\theta^* = 2 \frac{[p^+(\ell^-)p^-(\ell^+) - p^-(\ell^-)p^+(\ell^+)]}{m(\ell^+\ell^-)\sqrt{m^2(\ell^+\ell^-) + p_T^2(\ell^+\ell^-)}}$$

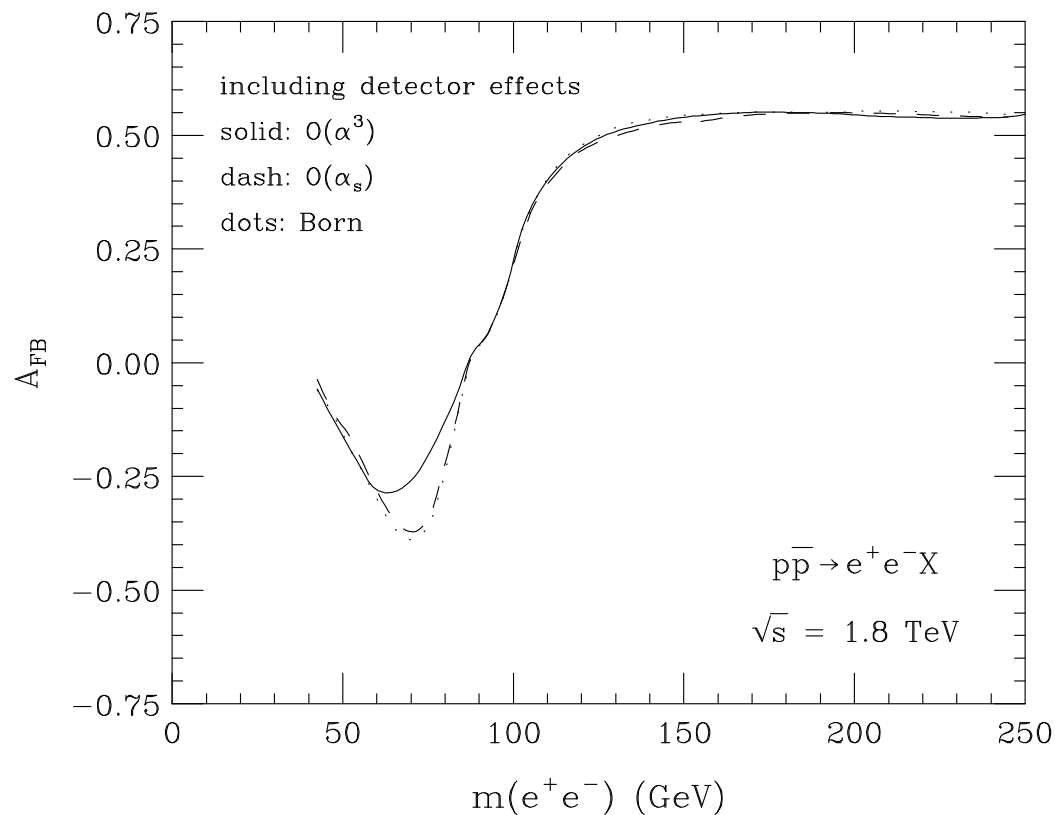
(Collins-Soper) with

$$p^\pm = \frac{1}{\sqrt{2}} (E \pm p_z)$$

☞ ie. the polar axis is the bisector of \mathbf{p}_p and $-\mathbf{p}_{\bar{p}}$, when they are boosted into the $\ell^+\ell^-$ rest frame

☞ works because, at the Tevatron, the direction of the incoming quark coincides most of the time with the p beam direction

- ☞ QED corrections significantly reduce A_{FB} in magnitude below the Z peak
- ☞ in this region they are more important than the $\mathcal{O}(\alpha_s)$ QCD corrections
- ☞ above the Z , both QED and QCD corrections are quite small



- Impact of radiative corrections on M_Z

- ☞ EWK radiative corrections have a profound impact on the Z mass extracted by experiment

- ☞ using the calculation by Berends and Kleiss, CDF and DØ found that $\mathcal{O}(\alpha)$ corrections shift M_Z by $\approx -100 \text{ MeV}$ ($\approx -300 \text{ MeV}$) for $Z \rightarrow e^+e^-$ ($Z \rightarrow \mu^+\mu^-$)

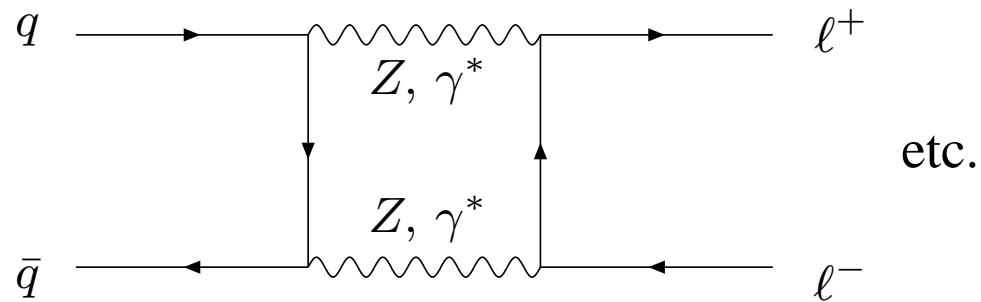
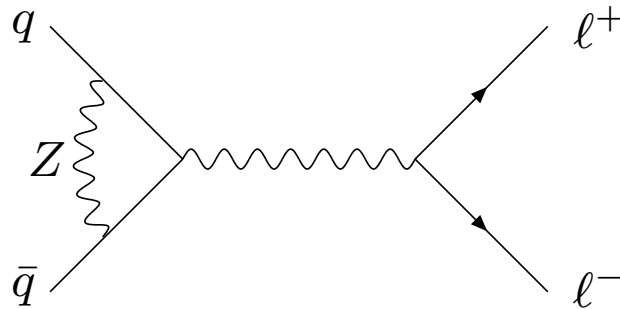
- ☞ The Z mass obtained from the complete $\mathcal{O}(\alpha^3)$ QED calculation is about **10 MeV smaller** than that obtained using Berends and Kleiss.

- ☞ almost all of the 10 MeV comes from the virtual and soft final state corrections; the contribution of ISR effects to the mass shift is negligible.

- ➔ the dependence of the Z mass extracted from experiment on the QED factorization scheme is negligible

- Weak Corrections in Z Boson Production

➡ Now add purely weak corrections



- to calibrate using LEP data one should use **exactly** the same theory input:

☞ include QCD corrections and $\mathcal{O}(g^4 M_t^2 / M_W^2)$ corrections

- results:

→ use CDF II cuts:

$$p_T(\ell) > 20 \text{ GeV}, \quad |\eta(\ell)| < 2.5$$

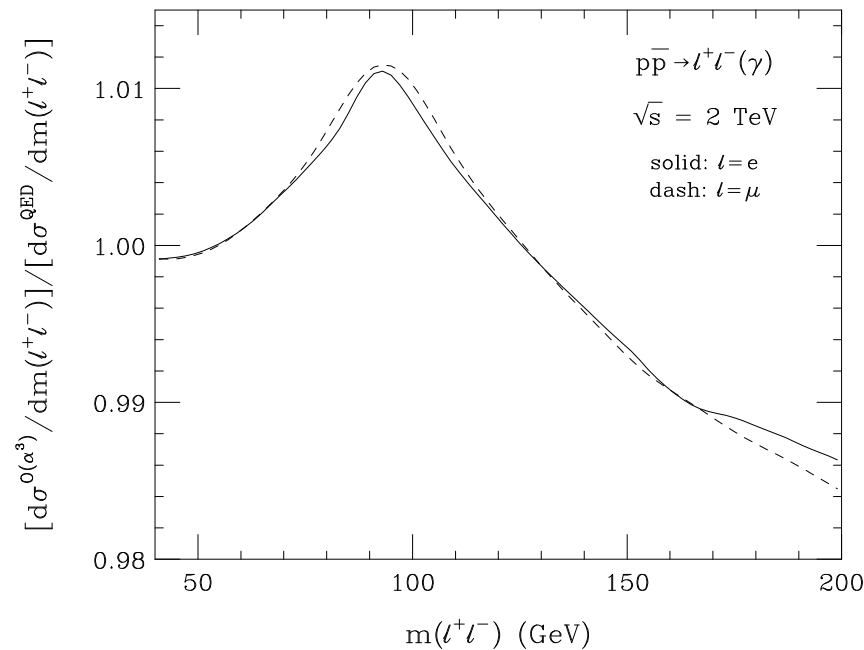
→ recombine electrons and photons as before

→ for muons use same cut on energy as before

- purely weak corrections **enhance** the cross section in the Z peak region ($75 \text{ GeV} < m(\ell\ell) < 105 \text{ GeV}$) by $\approx 1.0\%$.
- **recall**: QED corrections **reduce** the cross section in the Z peak region by **several percent**; the precise amount depends on cuts and lepton id. requirements

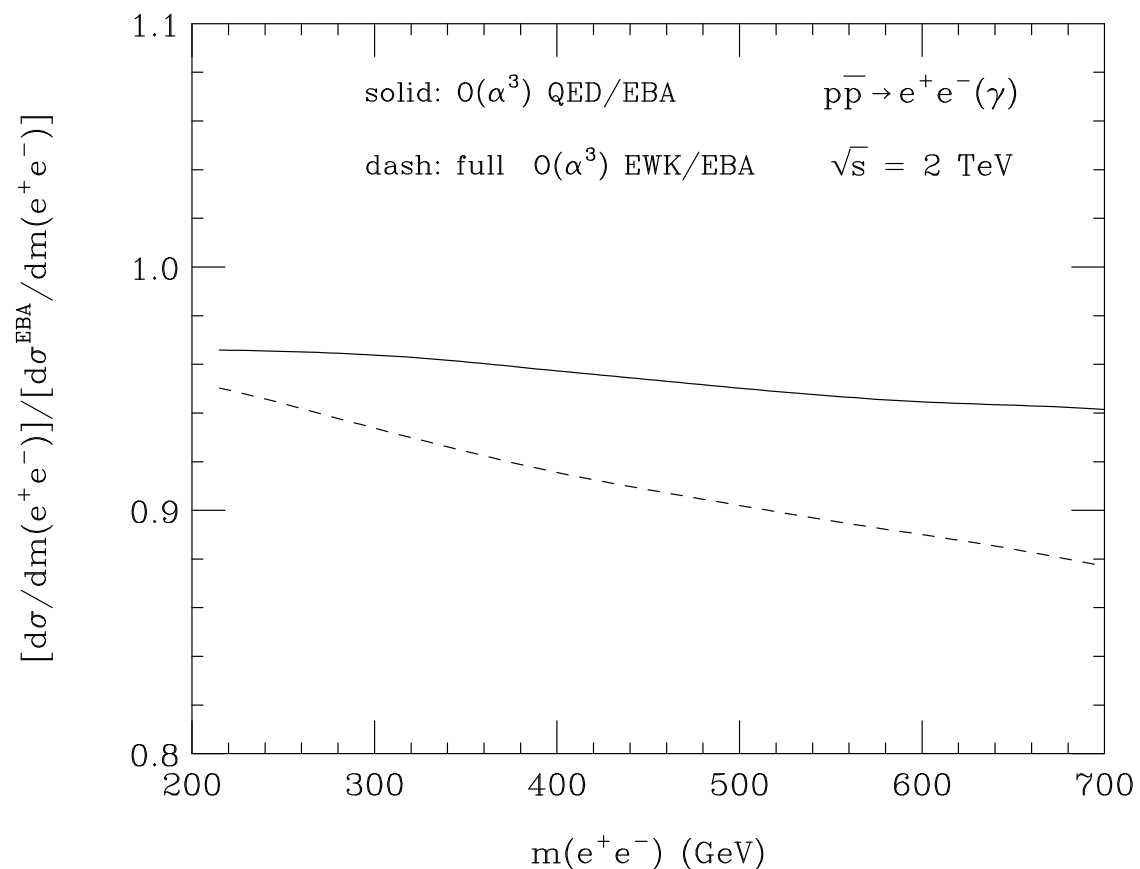
- For comparison: statistical uncertainty on $\sigma(Z \rightarrow \ell^+ \ell^-)$ for 2 fb^{-1} is **0.2%** per lepton channel
(of course, systematic uncertainties are (much) larger ...)

👉 low invariant mass region:



- 👉 notice the “pothole” at $\approx 160 \text{ GeV}$
- 👉 for $m(\ell\ell) > 120 \text{ GeV}$, ratio is < 1

👉 high invariant mass region:



- the weak corrections are more important than the QED corrections
- the weak corrections reduce the cross section

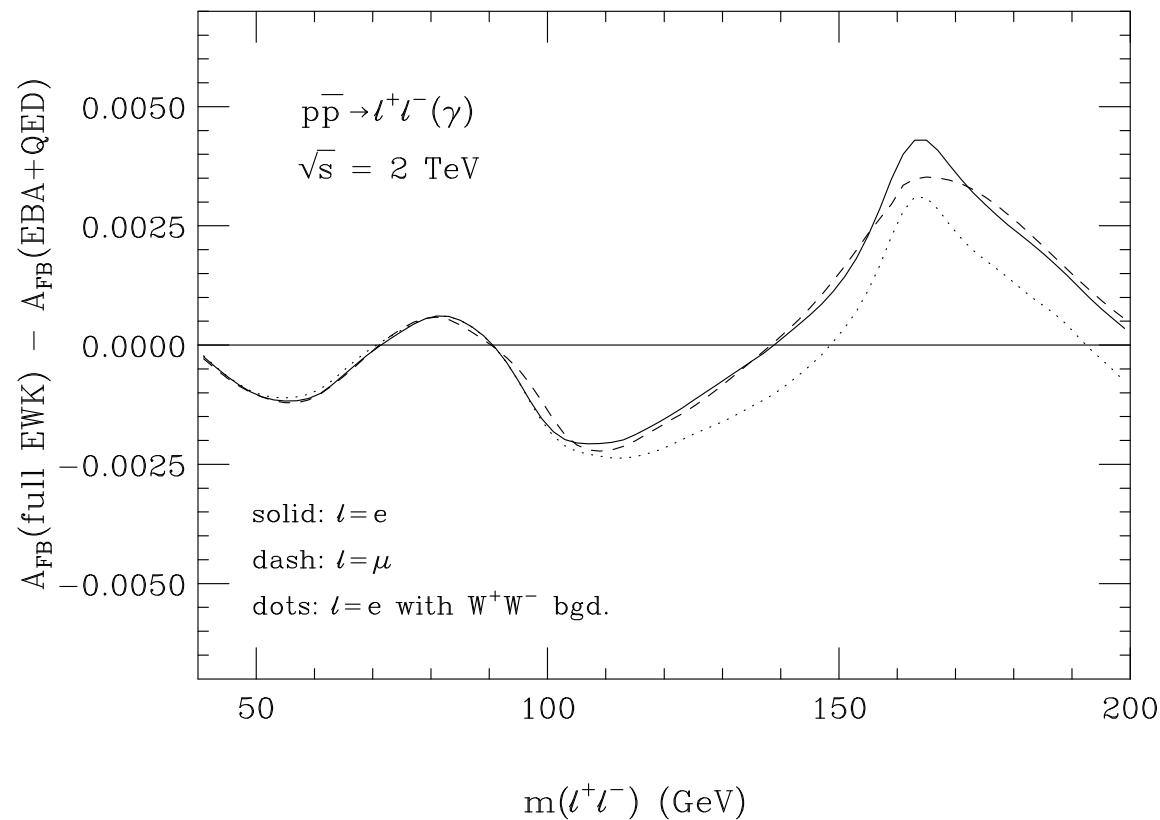
- For very large values of $m(\ell\ell)$ the weak corrections become significant:

$$\delta_{weak} \approx -9.5\% \quad \text{for} \quad m(\ell\ell) = 1000 \text{ GeV}$$

- **reason:** terms $\sim \alpha \log^2(\hat{s}/M_Z^2)$ from vertex and box corrections
 - ☞ need to resum? (**Kühn, Melles,...**)
 - ☞ certainly for the LHC this is necessary
- the large invariant mass region is interesting to probe for deviations from the SM (large extra dimensions, compositeness, etc.)

Weak Corrections to A_{FB} : A roller coaster

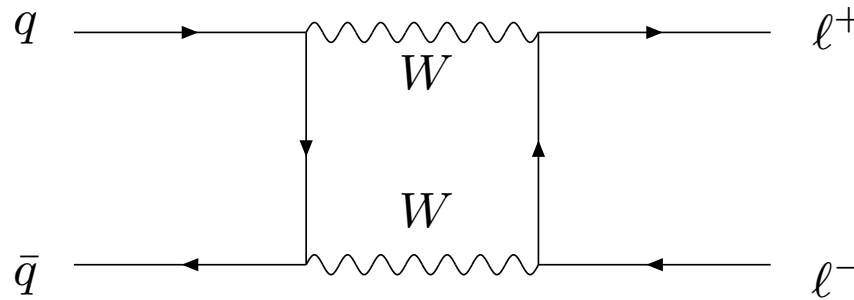
- low mass region (EBA: effective Born approximation
express σ_{Born} in terms of $\alpha(\hat{s})$, G_μ , and $\sin^2 \theta_{eff}^l$)



whow!

what is going on here?

- the peaks at ≈ 80 GeV and ≈ 160 GeV originate from the WW box diagram:



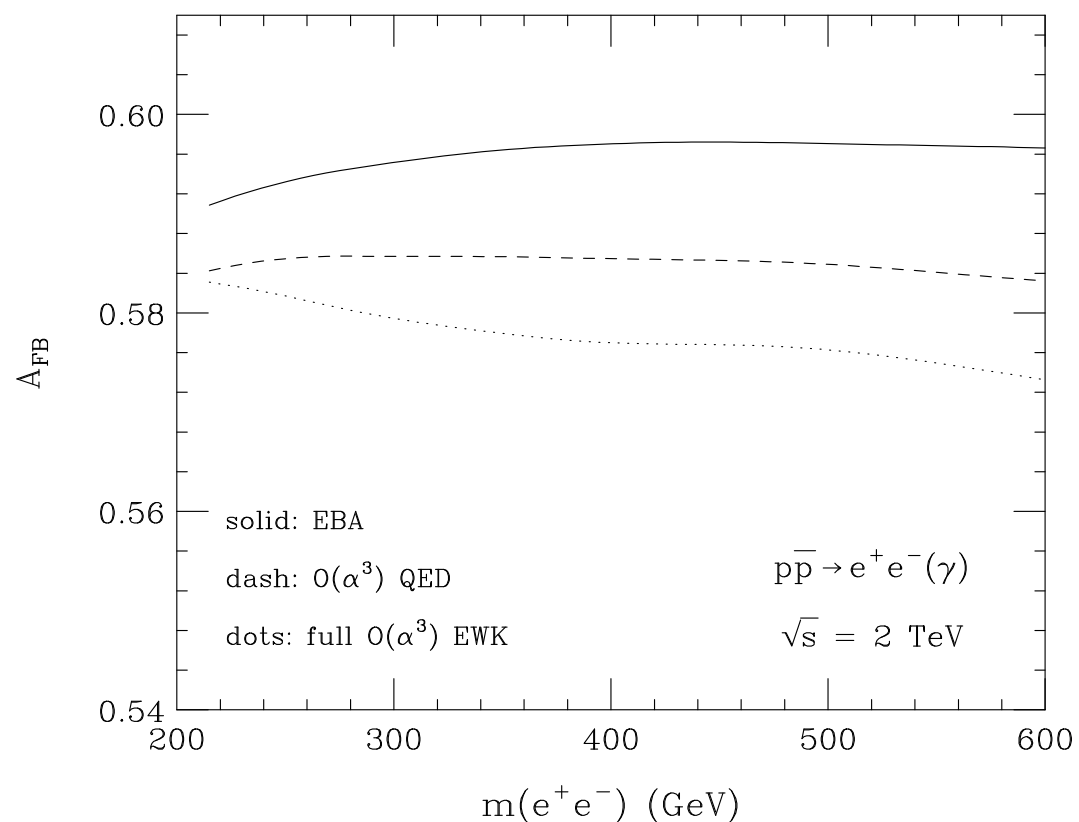
- $m(\ell\ell) = 80 \text{ GeV}$: one W goes on-shell
 $m(\ell\ell) = 160 \text{ GeV}$: both W 's are on-shell
 $m(\ell\ell) > 160 \text{ GeV}$ form factor develops large imaginary part
- the WW box effects are very pronounced in A_{FB} due to the $V - A$ nature of the $Wf\bar{f}$ coupling.

- Is the peak at $2M_W$ observable?
- for 8 fb^{-1} , the stat. uncertainty of A_{FB} for electrons (with the cuts specified above) in a 10 GeV bin centered at $m(\ell\ell) = 160 \text{ GeV}$ at the Tevatron is

$$\delta A_{FB} \approx 0.025$$

- variation of A_{FB} in this region due to purely weak corrections:
 $\approx 3 \times 10^{-3}$
 \rightarrow hard to observe
- experimental issue:
 need to subtract $W^+W^- \rightarrow \ell^+\ell^- \cancel{p}_T$ bgd. Two possibilities:
 $\rightarrow \cancel{p}_T$ veto ($\cancel{p}_T < 20 \text{ GeV}$)
 \rightarrow measure via $W^+W^- \rightarrow e^\pm \mu^\mp \cancel{p}_T$ and subtract

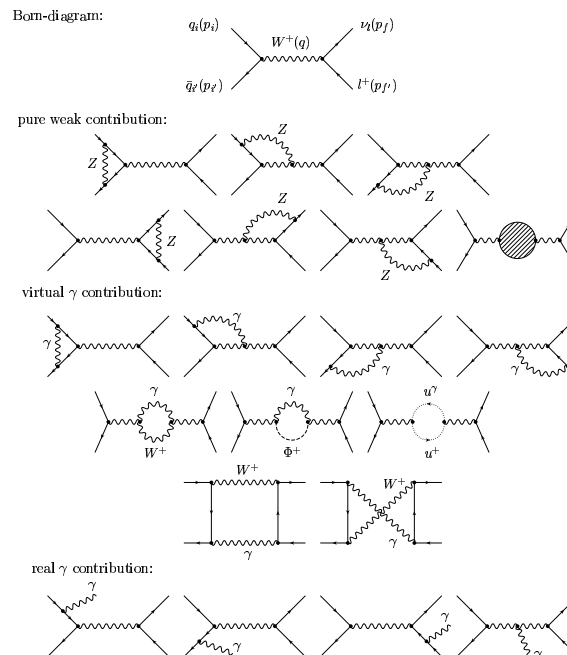
- high invariant mass region



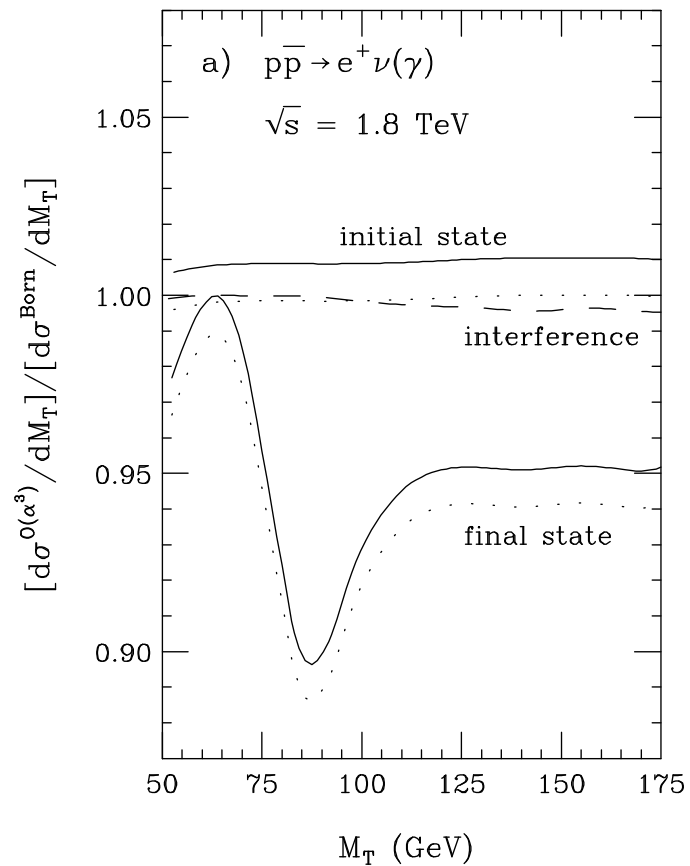
- ☞ both QED and weak corrections reduce A_{FB} and are of the same order
- ☞ the effect of the weak corrections is smaller than in the invariant mass distribution

3 – Electroweak Corrections to W Production

- Since the W is charged, the EWK corrections to W production **cannot** be separated into gauge invariant QED and purely weak corrections
 → need to take weak corrections into account right from start



- The EWK corrections can be arranged in such a way that they correspond to gauge invariant sets describing initial state, final state and interference contributions (Hollik, Wackerth)
- in the region around the W peak, one can evaluate form factors at $\hat{s} = M_W^2$ (pole approximation)
- technical details very similar to Z case
- phenomenological results (Tevatron, pole approximation)
 - ☞ use DØ-inspired detector model
 - ☞ observe significant corrections to transverse mass (M_T) distribution in Jacobian peak region



solid: QED like only

dots: QED like + modified weak

- The initial state QED like corrections are uniform and enhance the cross section by $\sim 1\%$

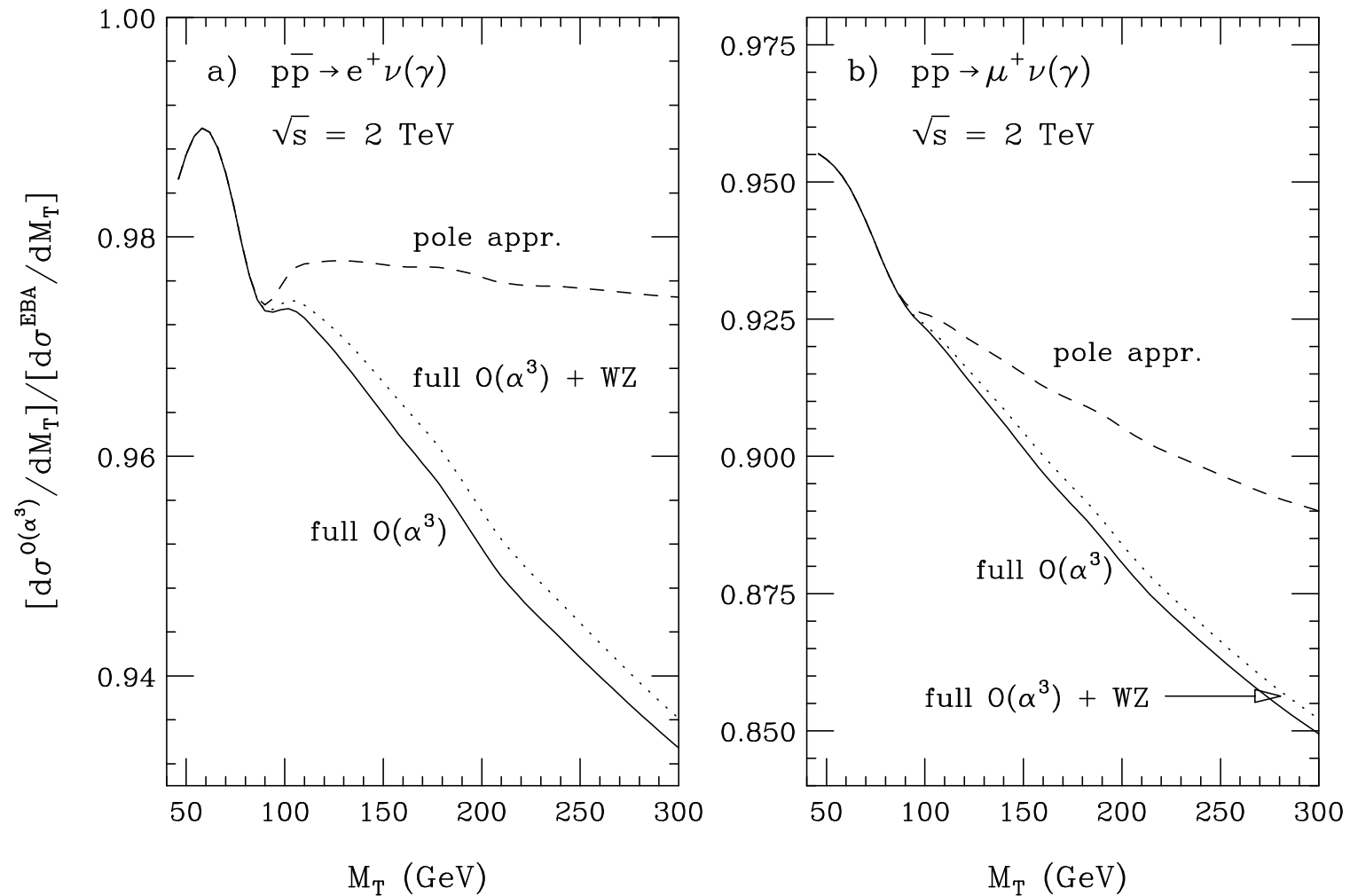
This is almost canceled by the modified weak initial state corrections

- the initial – final state interference terms are small and uniform
- the final state QED like corrections reduce the cross section by up to 10% (5%) for e (μ) final states

the final state modified weak corrections reduce the cross section by $\sim 1\%$

- As in the Z case, recombination of electron and photon momenta for small opening angles strongly reduces the effect of the EWK corrections, while in the μ case they become more pronounced

- the pole approximation breaks down away from the W peak region
- 👉 weak corrections become **large** above the peak (as in the Z case)



- impact of EWK corrections on W width measurement

☞ recall form of Breit-Wigner:

$$\frac{1}{(\hat{s} - M_W^2)^2 + \Gamma_W^2 \hat{s} / M_W^2}$$

☞ sensitivity to Γ_W comes from region where $\sqrt{\hat{s}} - M_W \sim \Gamma_W$

☞ cross section at peak scales like $1/\Gamma_W^2$ but this is washed out by detector resolution effects

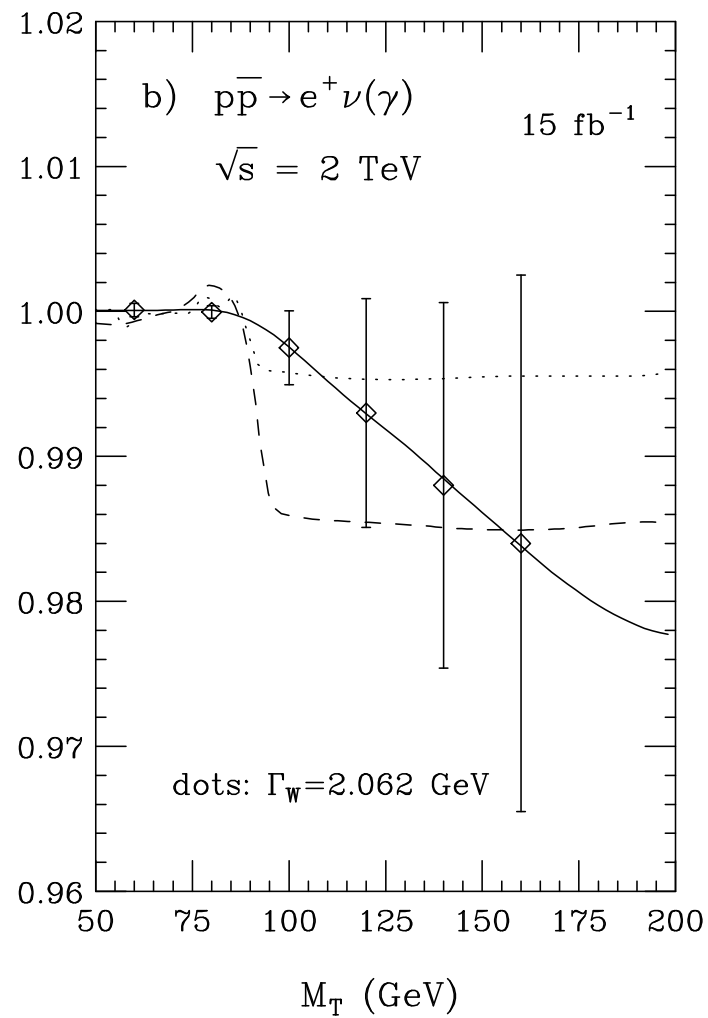
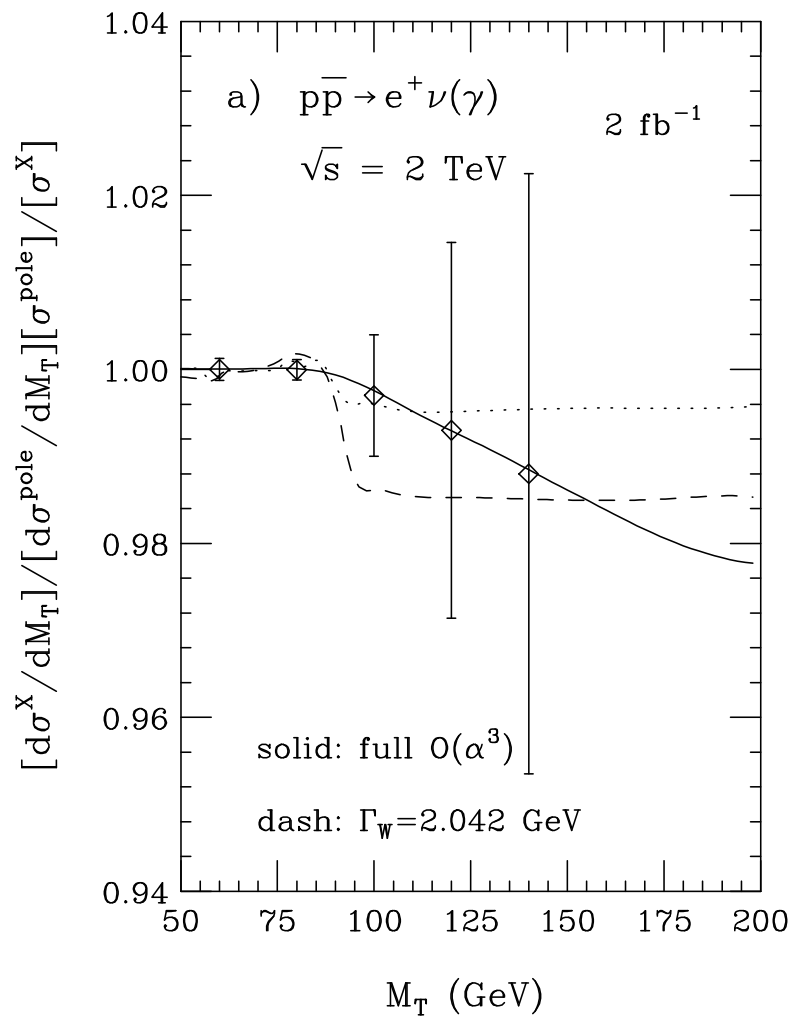
☞ σ_W scales like $1/\Gamma_W$

☞ ratio

$$\frac{\{[d\sigma/dM_T]/\sigma_W\}_{\Gamma_W^{SM}}}{\{[d\sigma/dM_T]/\sigma_W\}_{\Gamma_W}} \sim \frac{\Gamma_W}{\Gamma_W^{SM}}$$

at high values of M_T

- now suppose one compares data with pole approximation
 - ☞ compare shapes of M_T distributions by using normalized distributions
 - ☞ for input parameters chosen, $\Gamma_W^{SM} = 2.072 \text{ GeV}$
 - ☞ size of corrections ignored in pole approximation is of the same order as effects caused by non-SM values of Γ_W in the range accessible in Run II
- χ^2 fit: ignoring these corrections shifts Γ_W by -7.2 MeV if $M_T > 90 \text{ GeV}$ region is used for fit
 - ☞ this is not negligible compared with the expected precision in Run II ($50 \text{ MeV/channel/exp.}$)

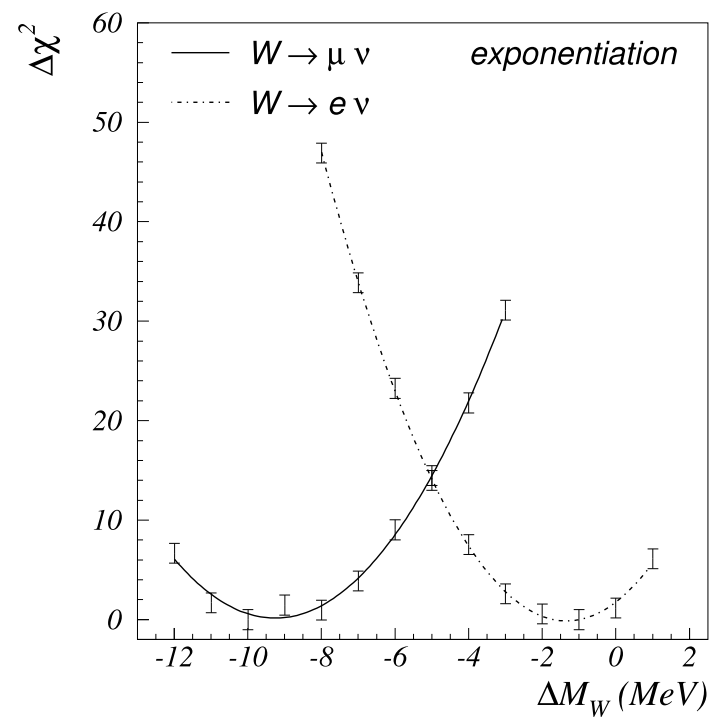
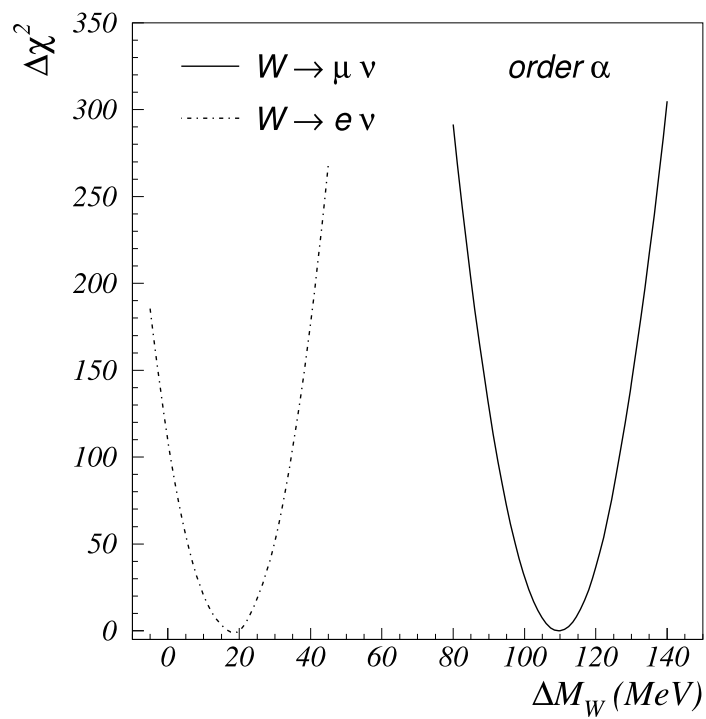


4 – Future Plans

- WGRAD and ZGRAD2 do not include QCD corrections
- need accurate simulation of W (and Z) recoil (W p_T) for W mass analysis
 - need unified generator which includes EWK and QCD corrections (including resummation)
- standard program to describe W/Z p_T : RESBOS (Balazs and Yuan)
 - ☞ RESBOS does not include EWK corrections
- both WGRAD/ZGRAD2 and RESBOS are standard tools for CDF/DØ analyses and have been interfaced with detector simulation software
- urgent need to unify/merge the two generators
 - ☞ in preparation (UB, D. Wackeroth, C.P. Yuan)

- final state photon radiation shifts W mass by $\mathcal{O}(100)$ MeV:
 - ☞ need to worry about multiple final state photon radiation
 - ☞ two recent papers on multi-photon radiation in W decays:
 - ➔ Jadach, Placzek, hep-ph/0302065
 - ➔ Montagna et al., hep-ph/0303102
- Jadach, Placzek (hep-ph/0302065):
 - ☞ use YFS exclusive exponentiation
 - ☞ currently only at parton level and for W production
 - ☞ we have started to collaborate with Jadach and Placzek to interface their calculation (**WINHAC**) with WGRAD / ZGRAD

- Montagna et al. (hep-ph/0303102):
 - ☞ calculate higher order real and virtual corrections using QED structure function approach
 - ☞ currently only for W production, and only final state corrections are incorporated (reasonable approximation for higher order photonic corrections)
 - ☞ calculated shift in M_W using simplified detector model:



- ➡ shift of M_W caused by multi-photon radiation is about 10% of that caused by one photon radiation
- ➡ **Note:** absolute value of shift caused by $\mathcal{O}(\alpha)$ corrections smaller than value observed by CDF/DØ due to simplified detector model
- ➡ larger effects expected in Z case: both final state leptons radiate photons

5 – Conclusions

- Calculations of the full $\mathcal{O}(\alpha)$ corrections to Z and W production now exist
- These calculations are essential ingredients for Run II and LHC precision electroweak measurements
- the electroweak corrections become large at high energies
- in the W case they will play a role in the determination of the W width from the tail of the transverse mass distribution
- need unified generator which includes resummed QCD corrections, $\mathcal{O}(\alpha)$ EWK corrections and resummed final state photon radiation effects